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Phil. Trans. R. Soc. Lond. A 1998 **356**, 1793-1821
doi: 10.1098/rsta.1998.0250

The Royal Society

Decoherence, einselection and the existential interpretation (the rough guide)

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The roles of decoherence and environment-induced superselection in the emergence of the classical from the quantum substrate are described. The stability of correlations between the einselected quantum pointer states and the environment allows them to exist almost as objectively as classical states were once thought to exist: there are ways of finding out what is the pointer state of the system which uses redundancy of its correlations with the environment, and which leave einselected states essentially unperturbed. This *relatively objective existence* of certain quantum states facilitates operational definition of probabilities in the quantum setting. Moreover, once there are states that 'exist' and can be 'found out', a 'collapse' in the traditional sense is no longer necessary—in effect, it has already happened. The role of the preferred states in the processing and storage of information is emphasized. The *existential interpretation* based on the relatively objective *existence* of stable correlations between the einselected states of observers' memory and in the outside universe is formulated and discussed.

Keywords: decoherence; einselection; environment-induced superselection

1. Introduction

The aim of the programme of decoherence and einselection (environment-induced superselection) is to describe consequences of the 'openness' of quantum systems to their environments and to study emergence of the effective classicality of some of the quantum states and of the associated observables. The purpose of this paper is to assess the degree to which this programme has been successful in facilitating the interpretation of quantum theory and to point out open issues and problems.

Much work in recent years has been devoted to the clarification and extension of the elements of the physics of decoherence and especially to the connections between measurements and environment-induced superselection (Zurek 1981, 1982, 1984, 1991; Joos & Zeh 1985; Walls *et al*. 1985; Caldeira & Leggett 1985; Unruh & Zurek 1989; Zeh 1993; Giulini *et al*. 1996). This has included studies of emergence of preferred states in various settings through the implementation of predictability sieves (Zurek 1993*a*; Zurek *et al*. 1993; Tegmark & Shapiro 1994; Gallis 1996), refinements of master equations and analysis of their solutions (Hu *et al*. 1992; Paz *et al*. 1993; Anglin & Zurek 1996; Anglin *et al*. 1997) and study of related ideas (such as consistent histories (Griffiths 1984, 1996; Gell-Mann & Hartle 1990; Omnès 1992), quantum trajectories and quantum state diffusion (Gisin & Percival 1992; Diósi *et al*. 1994; Carmichael 1993)). A useful counterpoint to these advances was provided by various applications, including quantum chaos (Dittrich & Graham 1990; Zurek & Paz 1994, 1995; Habib *et al*. 1998), einselection in the context of field theories

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and in Bose–Einstein condensates (Paz *et al*. 1993; Barnett *et al*. 1996; Wright *et al*. 1996) and, especially, by the interplay of the original information-theoretic aspects (Zurek 1981, 1982, 1983) of the environment-induced superselection approach with the recent explosion of research on quantum computation (Feynman 1986; Deutsch 1985; Lloyd 1993; DiVincenzo 1995; Bennett 1995; Williams & Clearwater 1997) and related subjects (Bennett 1995; Williams & Clearwater 1997; Schumacher 1995, 1996; Schumacher *et al*. 1996; Ekert & Jozsa 1996; DiVincenzo *et al*. 1998). Last but not least, the first controlled experiment aimed at investigating decoherence is now in place, carried out by Brune, Haroche, Raimond and their co-workers (Brune *et al*. 1996) and additional experiments may soon follow as a result of theoretical (Poyatos *et al*. 1996; Anglin *et al*. 1997; Bose *et al*. 1997; Cirac *et al*. 1998) and experimental (Monroe *et al*. 1996) developments.

In nearly all of the recent advances, the emphasis was on specific issues which could be addressed by detailed solutions of specific models. This attention to detail was necessary but may lead to the impression that practitioners of decoherence and einselection have lost sight of their original motivation—the interpretation of quantum theory. My aim here is to sketch 'the big picture', to relate the recent progress on specific issues to the overall goals of the programme. I shall therefore attempt to capture 'the whole' (or at least large parts of it), but in broad brush strokes. Special attention will be paid to issues such as the implications of decoherence for the origin of quantum probabilities, and to the role of information processing in the emergence of 'objective existence' which significantly reduces, and perhaps even eliminates, the role of the 'collapse' of the state vector.

In short, we shall describe how decoherence converts quantum entanglement into classical correlations and how these correlations can be used by the observer for the purpose of prediction. What will matter is then encoded in the *relations* between states (such as a state of the observer's memory and of the quantum systems). Stability of similar *correlations* with the environment allows observers to find out unknown quantum states without disturbing them. Recognition of this *relatively objective existence* of einselected quantum states and investigation of the consequences of this phenomenon are the principal goals of this paper. Relatively objective existence allows for the *existential interpretation* of quantum theory. Reduction of the wavepacket as well as the 'collapse' emerge as a consequence of the realization that the effectively classical states, including the states of the observer's memory, must exist over periods that are long compared to the decoherence time if they are to be useful as repositories of information.

It will be emphasized that while significant progress has been made since the environment-induced superselection programme was first formulated (Zurek 1981, 1982, 1984; Joos & Zeh 1985), much more remains to be done on several fronts which all have implications for the overarching question of interpretation. We can mention two such open issues right away: both the formulation of the measurement problem and its resolution through the appeal to decoherence require a universe split into systems. Yet, it is far from clear how one can define systems given an overall Hilbert space 'of everything' and the total Hamiltonian. Moreover, while the paramount role of information has been recognized, I do not belive that it has been, as yet, sufficiently thoroughly understood. Thus, while what follows is perhaps the most complete discussion of the interpretation implied by decoherence, it is still only a report of partial progress.

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2. Overview of the problem

While special relativity was discovered some 20 years before quantum mechanics in its modern guise was formulated, it has in a sense provided a model of what a new theory should be. As a replacement of Newtonian kinematics and dynamics, it was a seamless extension. In the limit of the infinite speed of light, $c \to \infty$, equations and concepts of the old theory were smoothly recovered.

When Bohr (1928), Heisenberg (1927), Born (1926) and Schrödinger (1935) struggled to understand the implications of quantum theory (see Wheeler & Zurek (1983) for reprints of many of the original papers), one can sense that they had initially expected a similar seamless extension of classical physics. Indeed, in specific cases, i.e. Bohr's correspondence, Heisenberg's uncertainty, Ehrenfest's theorem, such hopes were fulfilled in the appropriate limits (i.e. large quantum numbers, $\hbar \to 0$, etc.). However, Schrödinger's wavepackets did not travel along classical trajectories (except in the special case of the harmonic oscillator). Instead, they developed into delocalized non-classical superpositions. And the tempting $\hbar \to 0$ limit did not allow for the recovery of classical locality—it did not even exist, as the typical expression appearing in wavefunctions such as $\exp(\mathrm{i}xp/\hbar)$ is not even analytic as $\hbar \to 0$.

The culprit which made it impossible to recover classicality as a limiting case of quantum theory was at the very foundation of the quantum edifice: it was the quantum principle of superposition. It guarantees that any superposition of states is a legal quantum state. This introduced a whole Hilbert space $\mathcal{H}$ of possibilities, while only a small fraction of states in $\mathcal{H}$ can be associated with the classically allowed states, and superpositions of such states are typically flagrantly non-classical. Moreover, the number of possible non-classical states in the Hilbert space increases exponentially with its dimensionality, while the number of classical states increases only linearly. This divergence (which is perhaps the key of the ingredients responsible for the exponential speed-up of quantum computations (Feynman 1986; Deutsch 1985; Lloyd 1993; DiVincenzo 1995; Bennett 1995; Williams & Clearwater 1997)) is a measure of the problem. Moreover, it can be argued that it is actually exacerbated in the $\hbar \to 0$ limit, as the dimensionality of the Hilbert space (of say, a particle in a confined phase space) increases with $1/\hbar$ to some power.

The first resolution (championed by Bohr (1928)) was to outlaw 'by fiat' the use of quantum theory for the objects which were classical. This 'Copenhagen interpretation' (CI) had several flaws: it would have forced quantum theory to depend on classical physics for its very existence. It would have also meant that neither quantum nor classical theory was universal. Moreover, the boundary between them was never clearly delineated (and, according to Bohr, had to be 'movable' depending on the whims of the observer). Last but not least, with the collapse looming on the quantum–classical border, there was little chance for a seemless extension.

By contrast, Everett's 'many worlds' interpretation (MWI) (Everett 1957) refused to draw a quantum–classical boundary. Superposition principle was the 'law of the land' for the universe as a whole. Branching wave functions described alternatives, all of which were realized in the deterministic evolution of the universal state vector.

The advantage of Everett's original vision was to reinstate quantum theory as a key tool in search of its own interpretation. The disadvantages (which were realized only some years later, after the original proposal became more widely known) include (i) the ambiguity of what constitutes the 'branches' (i.e. specification of which of the states in the Hilbert spaces containing all of the conceivable superpositions are classically 'legal') and (ii) re-emergence of the questions about the origin of probabilities (i.e. the derivation of Born's formula). Moreover, (iii) it was never clear how to reconcile unique experiences of observers with the multitude of alternatives present in the MWI wavefunction.

3. Decoherence and einselection

Decoherence is a process of continuous measurement-like interaction between the system and an (external or internal) environment. Its effect is to invalidate the superposition principle in the Hilbert space of an open system. It leads to very different stability properties for various pure states. Interaction with the environment destroys the vast majority of the superpositions quickly, and, in the case of macroscopic objects, almost instantaneously. This leads to negative selection which in effect bars most of the states and results in singling out of a preferred stable subset of the einselected pointer states.

Correlations are both the cause of decoherence and the criterion used to evaluate the stability of the states. Environment correlates (or, rather, becomes entangled) with the observables of the system while 'monitoring' them. Moreover, stability of the correlations between the states of the system monitored by their environment and of some other 'recording' system (i.e. an apparatus or a memory of an observer) is a criterion of the 'reality' of these states. Hence, we shall often talk about *relatively objective existence* of states to emphasize that they are really defined only through their correlations with the state of the other systems, as well as to remind the reader that these states will never be quite as 'rock solid' as classical states of a stone or a planet were (once) thought to be.

Transfer of a single bit of information is a single 'unit of correlation', whether in communication, decoherence or in measurement†. It suffices to turn a unit of *quantum* correlation (i.e. entanglement, which can be established in the course of the (pre)measurement—like an interaction between two one-bit systems) into a *classical* correlation.

This process is illustrated in figure 1 with a 'bit-by-bit' measurement (Zurek 1981)—a quantum controlled-not (or a C-NOT). An identical C-NOT controlled by the previously passive 'target' bit (which played the role of the apparatus pointer in the course of the initial correlation (figure 1*a*)) and a bit 'somewhere in the environment' represents the process of decoherence. Now, however, the former apparatus (target) bit plays a role in the control. As a result, a pure state of the system,

$$|\sigma\rangle = \alpha|0\rangle + \beta|1\rangle, \tag{3.1}$$

is 'communicated' by first influencing the state of the apparatus:

$$|\Phi_{\mathcal{SA}}(0)\rangle = |\sigma\rangle_{\mathcal{S}}|0\rangle_{\mathcal{A}} \to \alpha|00\rangle_{\mathcal{SA}} + \beta|11\rangle_{\mathcal{SA}} = |\Phi_{\mathcal{SA}}(t_1)\rangle, \tag{3.2}$$

† It is no accident that the set-ups used in modern treatments of quantum communication channels (Bennett 1995; Williams & Clearwater 1997; Schumacher 1995, 1996; Schumacher *et al.* 1996) bear an eerie resemblance to the by now 'old hat' system–apparatus–environment 'trio' used in the early discussions of environment-induced superselection (Zurek 1981, 1982). The apparatus $\mathcal{A}$ is a member of this trio which is supposed to preserve, in the preferred pointer basis, the correlation with the state of the system $\mathcal{S}$ with which it is initially entangled. Hence, $\mathcal{A}$ is a 'communication channel'.

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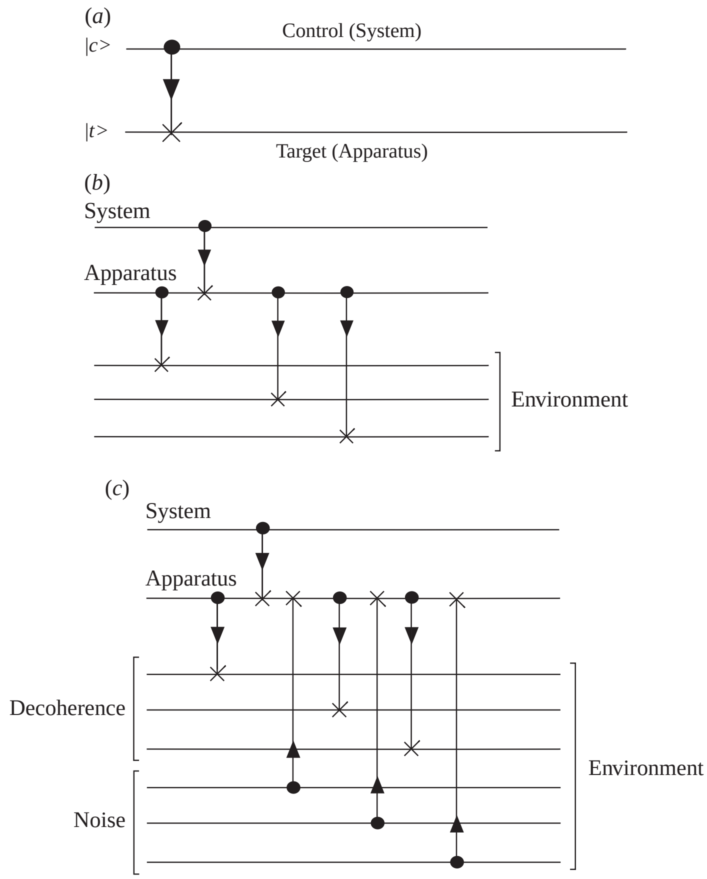

Figure 1. Information transfer in measurements and in decoherence. (*a*) Controlled not (C-NOT) as an elementary bit-by-bit measurement. Its action is described by the 'truth table' according to which the state of the target bit (apparatus memory in the quantum measurement vocabulary) is 'flipped' when the control bit (measured system) is $|1\rangle$ and untouched when it is $|0\rangle$ (equation (3.2)). This can be accomplished by the unitary Schrödinger evolution (see Zurek (1981, 1983) and Deutsch (1985) for the information theoretic discussion). (*b*) Decoherence process 'caricatured' by means of C-NOTs. Pointer state of the apparatus (and, formerly, target bit in the premeasurement (*a*)) now acts as a control in the continuous monitoring by the C-NOTs of the environment. This continuous monitoring process is symbolically 'discretized' here into a sequence of C-NOTs, with the state of the environment assuming the role of the multibit target. Monitored observable of the apparatus—its pointer observable—is in the end no longer entangled with the system, but the classical correlation remains. Decoherence is associated with the transfer of information about the to-be-classical observables to the environment. Classically, such information transfer is of no consequence. In quantum physics it is, however, absolutely crucial, as it is responsible for the effective classicality of certain quantum observables, and for the relatively objective existence of preferred pointer states. (*c*) Noise is a process in which a pointer observable of the apparatus is perturbed by the environment. Noise differs from the purely quantum decoherence—now the environment acts as a control, and the C-NOTs which represent it carry information in the direction opposite to the decoherence C-NOTs. Usually, both decoherence and noise are present. Preferred pointer observables and the associated pointer states are selected so that the noise is minimized.

and then by spreading that influence to the environment:

$$|\Psi_{\mathcal{SAE}}(t_1)\rangle = (\alpha|00\rangle + \beta|11\rangle)|0\rangle \rightarrow \alpha|000\rangle + \beta|111\rangle = |\Psi_{\mathcal{SAE}}(t_2)\rangle \ . \qquad (3.3)$$

Above, we have dropped the indices $\mathcal{SAE}$ for individual qubits (which would have appeared in the obvious order).

After the environment is traced out, only the correlation with the pointer basis of the apparatus (i.e. the basis in which the apparatus acts as a control) will survive (Zurek 1981, 1982, 1983, 1984):

$$\rho_{\mathcal{SA}}(t_2) = |\alpha|^2|00\rangle\langle 00| + |\beta|^2|11\rangle\langle 11|. \qquad (3.4)$$

Thus, the apparatus plays the role of the communication channel (memory) (i) through its ability to retain correlations with the measured system, but also, (ii) by 'broadcasting' these correlations into the environment which is the source of decoherence (see figure 1*b*). Such broadcasting of quantum correlations makes them, and the observables involved in broadcasting, effectively classical (Barnum *et al*. 1996).

The ability to retain correlations is the defining characteristic of the preferred 'pointer' basis of the apparatus. In simple models of measurement *cum* decoherence, the selection of the preferred basis of the apparatus can be directly tied to the form of the interaction with the environment. Thus, an observable $\hat{O}$ which commutes with the complete (i.e. self-, plus the interaction with the environment) Hamiltonian of the apparatus,

$$[\hat{H}_{\mathcal{A}} + \hat{H}_{\mathcal{AE}}, \hat{O}] = 0, \qquad (3.5)$$

will be the pointer observable. This criterion can be fulfilled only in the simplest cases: typically, $[\hat{H}_{\mathcal{A}}, \hat{H}_{\mathcal{AE}}] \neq 0$, hence equation (3.5) cannot be satisfied exactly.

In more realistic situations one must therefore rely on more general criteria to which we have alluded above. One can start by noting that the einselected pointer basis is best at retaining correlations with the external stable states (such as pointer states of other apparatus or record states of the observers). The predictability sieve (Zurek 1993*a*; Zurek *et al*. 1993; Tegmark & Shapiro 1994; Gallis 1996) is a convenient strategy to look for such states. It retains pure states which produce the least entropy over a period of time that is long compared to the decoherence time-scale. Such states avoid entanglement with the environment and, thus, can preserve correlations with the similarly selected states of other systems. In effect, the predictability sieve can be regarded as a strategy to select stable correlations.

A defining characteristic of the *reality* of a state is the possibility of finding out what it is and yet leaving it unperturbed. This criterion of *objective existence* is of course satisfied in classical physics. It can be formulated operationally by devising a strategy which would let an observer who was previously unaware of the state find out what it is and later verify that the state was (i) correctly identified, and (ii) not perturbed. In quantum theory, this is not possible to accomplish with an *isolated* system. Unless the observer knows in advance what observables commute with the state of the system, he will in general end up repreparing the system through a measurement employing 'his' observables. This would violate condition (ii) above. So—it is said—quantum states do not *exist objectively*, since it is impossible to find out what they are without, at the same time, 'remolding them' with the questions posed by the measurement (Wheeler 1983).

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Einselection allows states of an open quantum system to pass the 'existence test' in several ways. The observer can, for example, measure properties of the Hamiltonian which generates the evolution of both the system and the environment. Einselection determines that pointer states will appear on the diagonal of the density matrix of the system. Hence, the observer can know beforehand what (limited) set of observables can be measured with impunity. He will be able to select measurement observables that are already monitored by the environment. Using a set of observables codiagonal in the Hilbert space of the system with the einselected states, he can then perform a non-demolition measurement to find out what the state is without perturbing it.

A somewhat indirect strategy which also works involves monitoring the environment and using a *fraction* of its state to infer the state of the system. This may not always be feasible, but this strategy is worth noting since it is the one universally employed by us, the real observers. Photons are one of the most pervasive environments. We gather most of our information by intercepting a small fraction of that environment. Different observers agree about reality based on a consensus reached in this fashion.

That such a strategy is possible can be readily understood from the C-NOT 'caricature' of decoherence in figure 1. The einselected control observables of the system, or of the apparatus, are redundantly recorded in the environment. One can then 'read them off' many times (even if each read-off may entail erasure of a part of the information from the environment) without interacting directly with the system of interest.

It is important to emphasize that the relatively objective existence is attained at the price of partial ignorance. The observer should *not* attempt to intercept *all* of the environment state (which may be entangled with the system and, hence, could be used to redefine its state by sufficiently outrageous measurement (Carmichael *et al*. 1993)). Objective existence is objective only because part of the environment has 'escaped' with the information about the state of the system and can continue to serve as a 'witness' to what has happened. It is also important that the fraction of the environment which escapes should not matter, except in the two limits when the observer can intercept all of the relevant environment (the entanglement limit), and in the case when the observer simply does not intercept enough (the ignorance limit).

This robustness of the preferred (einselected) observables of the system can be quantified through redundancy (Zurek 1983), in a manner reminiscent of the recent discussions of the error-correction strategies (see, for example, DiVincenzo *et al*. (1998), and references therein). Consider, for example, a correlated state

$$|\Psi_{\mathcal{SE}}\rangle = (|0\rangle_{\mathcal{S}}|000\rangle_{\mathcal{E}} + |1\rangle_{\mathcal{S}}|111\rangle_{\mathcal{E}})/\sqrt{2} \tag{3.6}$$

which could have arisen from a sequence of three system–environment C-NOTs. All errors afflicting individual qubits of the environment can be classified by associating them with Pauli matrices acting on individual qubits of the environment. We can now inquire about the number of errors which would destroy the correlation between various observables of the system and the state of the environment. It is quite obvious that the states $\{|0\rangle_{\mathcal{S}}, |1\rangle_{\mathcal{S}}\}$ are in this sense more robustly correlated with the environment than the states $\{|+\rangle_{\mathcal{S}}, |-\rangle_{\mathcal{S}}\}$ obtained by Hadamard transform:

$$|\Psi_{\mathcal{SE}}\rangle = |+\rangle_{\mathcal{S}}(|000\rangle_{\mathcal{E}} + |111\rangle_{\mathcal{E}})/\sqrt{2} + |-\rangle_{\mathcal{S}}(|000\rangle_{\mathcal{E}} - |111\rangle_{\mathcal{E}})/\sqrt{2}. \tag{3.7}$$

For, a phase flip of any of the environment bits would destroy the ability of the observer to infer the state of the system in the $\{|+\rangle_{\mathcal{S}}, |-\rangle_{\mathcal{S}}\}$ basis. By contrast, a majority vote in a $\{|0\rangle_{\mathcal{E}}, |1\rangle_{\mathcal{E}}\}$ basis would still yield a correct answer concerning $\{|0\rangle_{\mathcal{S}}, |1\rangle_{\mathcal{S}}\}$ if *any* single error afflicted the state of the environment. Moreover, when there are N bits in the environment, $\frac{1}{2}N-1$ errors can be in principle still tolerated in the $\{|0\rangle_{\mathcal{S}}, |1\rangle_{\mathcal{S}}\}$ basis, but in the $\{|+\rangle_{\mathcal{S}}, |-\rangle_{\mathcal{S}}\}$ basis a simple phase flip continues to have disastrous consequences.

When we assume (as seems reasonable) that the probability of errors increases with the size of the environment (N), so that the 'specific error rate' (i.e. the probability of an error per bit of environment per second) is fixed, it becomes clear that the stability of pointer states is purchased at the price of the instability of their Hadamard–Fourier conjugates. This stabilization of certain observables at the expense of their conjugates may be achieved through either the deliberate amplification or as a consequence of accidental environmental monitoring, but in any case it leads to redundancy (Zurek 1983).

This redundancy may be quantified by counting the number of 'flips' applied to individual environment qubits which 'exchange' the states of the environment corresponding to the two states of the system. Thus, we can compute the redundancy distance d between the record states of the environment in the case corresponding to the two system states ϕ, ψ given by $\{|0\rangle_{\mathcal{S}}, |1\rangle_{\mathcal{S}}\}$ in the decomposition of equation (3.6):

$$d(\phi, \psi) = N,$$

while in the case of the complementary observable with ϕ, ψ given by $\{|+\rangle_{\mathcal{S}}, |-\rangle_{\mathcal{S}}\}$:

$$d(\phi, \psi) = 1.$$

Or, in general, redundancy distance

$$d(\phi, \psi) = \min(n_x + n_y + n_z) \tag{3.8}$$

is the least total number of 'flips', where n_x, n_y and n_z are the numbers of $\hat{\sigma}_x$, $\hat{\sigma}_y$, and $\hat{\sigma}_z$ operations required to convert the state of the environment correlated with $|\phi\rangle$, which is given, up to the normalization constant, by

$$|\mathcal{E}_\phi\rangle = \langle\phi|\Psi_{\mathcal{SE}}\rangle, \tag{3.9}$$

with the similarly defined $|\mathcal{E}_\psi\rangle$.

Redundancy defined in this manner is indeed a measure of distance, since it is (i) non-negative,

$$d(\phi, \psi) \geqslant 0; \tag{3.10}$$

(ii) symmetric,

$$d(\phi, \psi) = d(\psi, \phi); \tag{3.11}$$

and (iii) satisfies the triangle inequality,

$$d(\phi, \psi) + d(\psi, \gamma) \geqslant d(\phi, \gamma), \tag{3.12}$$

as the reader should be able to establish without difficulty.

In the simplest models which satisfy the commutation condition (equation (3.5)), the most predictable set of states will consist of the eigenstates of the pointer observable $\hat{O}$. They will not evolve at all and, hence, will be perfect memory states as well

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as the most (trivially) predictable classical states. In the more general circumstances the states which commute with $\hat{H}_{\mathcal{SE}}$ at one instant will be rotated (into their superpositions) at a later instant with the evolution generated by the self-Hamiltonian $\hat{H}_{\mathcal{S}}$. Thus, a near-zero entropy production at one instant may be 'paid for' by an enormous entropy production rate a while later. An example of this situation is afforded by a harmonic oscillator, where the dynamical evolution periodically 'swaps' the state vector between its position and momentum representation, and the two representations are related to each other by a Fourier transformation. In that case the states which are most immune to decoherence in the long run turn out to be the fixed points of the 'map' defined by the Fourier transformation. Gaussians are the fixed points of the Fourier transformation (they remain Gaussian). Hence, coherent states which are unchanged by the Fourier transform are favoured by decoherence (Zurek 1993*a*; Zurek *et al*. 1993; Tegmark & Shapiro 1994; Gallis 1996).

In more general circumstances entropy production may not be minimized by an equally simple set of states, but the lessons drawn from the two extreme examples discussed above are nevertheless relevant. In particular, in the case of systems dominated by the environmental interaction, the Hamiltonian $\hat{H}_{\mathcal{SE}}$ will have a major say in selecting the preferred basis, while in the underdamped case of the near-reversible 'Newtonian' limit, approximately Gaussian wavepackets localized in both position and momentum will be optimally predictable, leading to the idealization of classical trajectories. In either case, einselection will pin-point the stable set of states in the Hilbert space. These pointer states will be stable, but their superpositions will deteriorate into pointer-state mixtures rather quickly, on the decoherence time-scale, which tends to happen very much faster than relaxation (Zurek 1984).

This eventual diagonality of the density matrix in the einselected basis is a by-product, an important symptom, but not the essence of decoherence. I emphasize this because diagonality of $\rho_{\mathcal{S}}$ in some basis has been occasionally (mis)interpreted as a key accomplishment of decoherence. This is misleading. Any density matrix is diagonal in some basis. This has little bearing on the interpretation. Well-known examples of such accidental diagonality are the unit density matrix (which is diagonal in every basis) and the situation where $\rho_{\mathrm{A}\cup\mathrm{B}} = p\rho_{\mathrm{A}} + (1-p)\rho_{\mathrm{B}}$ describes a union of two ensembles A and B with density matrices ρ_{A} and ρ_{B} which are not codiagonal (i.e. $[\rho_{\mathrm{A}}, \rho_{\mathrm{B}}] \neq 0$). In either of these two cases states which are on the diagonal of $\rho_{\mathrm{A}\cup\mathrm{B}}$ are in effect a mathematical accident and have nothing to do with the physical reality.

Einselection chooses a preferred basis in the Hilbert space in recognition of its predictability. That basis will be determined by the dynamics of the open system in the presence of environmental monitoring. It will often turn out that it is overcomplete. Its states may not be orthogonal, and, hence, they would never follow from the diagonalization of the density matrix.

Einselection guarantees that only those ensembles which consist of a mixture of pointer states can truly 'exist' in the quasi-classical sense. That is, individual members of such ensembles are already immune to the measurement of pointer observables. These remarks cannot be made about an arbitrary basis which happens to diagonalize ρ, but are absolutely essential if the quantum system is to be regarded as effectively classical.

It is useful to contrast decoherence with the more familiar consequence of interactions with the environment—the noise. Idealized decoherence (e.g. the case of equation (3.5)) has absolutely no effect on the observable of interest. It is caused by the environment carrying out a continuous 'non-demolition measurement' on the pointer observable $\hat{O}$. Thus, decoherence is caused by the system observables effecting the environment and by the associated transfer of information. Decoherence is, in this sense, a purely quantum phenomenon; information transfers have no effect on classical states.

Noise, by contrast, is caused by the environment disturbing an observable. It is, of course, familiar in the classical context. The distinction between the two is illustrated in figure 1*c*, in the C-NOT language we have adopted previously.

Astute readers will point out that the distinction between noise and decoherence is a function of the observable in terms of which C-NOT is implemented. This is because a quantum C-NOT is, in contrast with its classical counterpart, a 'two-way street'. When the Hadamard transform, $|\pm\rangle = (|0\rangle \pm |1\rangle)/\sqrt{2}$, is carried out, control and target swap their functions. Thus, loosely speaking, as the information about the states $\{|0\rangle, |1\rangle\}$ travels in one direction, the information about the relative phase (which is encoded in their Hadamard transforms) travels the other way. Thus, in quantum gates the direction of the information flow depends on the states which are introduced at the input.

Typically, both noise and decoherence are present. One can reinterpret the predictability sieve (Zurek 1993*a*) we have mentioned before as a search for the set of states which maximizes the 'control' role of the system, while simultaneously minimizing its 'target' role. Eigenstates of the pointer observable are a solution. The phases between them are a 'victim' of decoherence and are rapidly erased by the interaction with the environment.

4. Probabilities

The classical textbook by Gnedenko (1962) distinguishes the following three ways of defining probability.

(i) Definitions which introduce probability as a quantitative measure of the *degree of certainty* of the observer.

(ii) 'Standard† definitions', which originate from the more primitive concept of *equal likelihood* (and which can be traced to the original applications of probability in gambling).

(iii) *Relative frequency* definitions, which attempt to reduce probability to a frequency of occurrence of an event in a large number of trials.

In the context of the interpretation of quantum theory, the last of these three definitions has been invoked most often in attempts to derive probabilities from the universal quantum dynamics (Graham 1970; Hartle 1968; Fahri *et al*. 1989; Kent 1990). The argument starts with an ensemble of identical systems (e.g. spin-$\frac{1}{2}$ systems) in a pure state and a definition of a relative frequency operator for that ensemble. The intended role of the relative frequency operator was to act as a quantum equivalent of a classical 'counter', but in effect it was always a meta-observable of the whole ensemble, and, thus, it could not have been associated with the outcomes of measurements of the individual members of the ensemble.

† 'Classical' is a more often used adjective. We shall replace it with 'standard' to avoid confusion with the other kind of classicality discussed here.

A useful insight into relative frequency observables is afforded by the physically transparent example of Fahri *et al*. (1989). They consider an ensemble of spin-$\frac{1}{2}$ 'magnets', all in an identical state, aligned with some axis $\boldsymbol{a}$. A relative frequency observable along some other direction $\boldsymbol{b}$ would correspond to a measurement of a deflection of the object with a known mass and with a whole ensemble of spins attached to it by a (meta) Stern–Gerlach apparatus with a field gradient parallel to $\boldsymbol{b}$. The angle of deflection would then be proportional to $\boldsymbol{a}\cdot\boldsymbol{b}$, and $\frac{1}{2}(1+\boldsymbol{a}\cdot\boldsymbol{b})$ would be an eigenvalue of the frequency operator. However, none of the spins individually would be required to choose its 'answer'.

This approach is of interest as it sheds light on the properties of collective observables in quantum physics, but it does not lend itself to the required role of supplying the probability interpretation in the MWI context. A true 'frequency' with a classical interpretation cannot be defined at a level which does not allow 'events'—quantum evolutions which lead to objectively existing states—to be associated with the individual members of that ensemble. This criticism has been made already, in a somewhat different guise, by several authors (see Kent (1990), and references therein). The problem is in part traceable to the fact that the relative frequency observables do not eliminate superpositions between the branches of the universal wavefunction and do not even define what these branches are.

Decoherence has obvious implications for the probability interpretation. The reduced density matrix ρ, which emerges following the interaction with the environment, and a partial trace will always be diagonal in the *same* basis of einselected pointer states $\{|i\rangle\}$. These states help define elementary 'events'. Probabilities of such events can be inferred from their coefficients in ρ, which have the desired 'Born's rule' form.

Reservations about this straightforward reasoning have been expressed. Zeh (1997) has noted that interpreting coefficients of the diagonal elements of a density matrix as probabilities may be circular. Here we shall therefore examine this problem more closely and prove operational equivalence of two ensembles—the original ensemble $\mathbf{o}$ associated with the set of identical decohering systems, and an artificial ensemble $\boldsymbol{a}$, constructed to have the same density matrix ($\rho_{\mathbf{o}}=\rho_{\boldsymbol{a}}$), but for a much more classical reason, which allows for a straightforward interpretation in terms of relative frequencies. This will also shed light on the sense in which the origin of quantum probabilities can be associated with the ignorance of observers.

The density matrix alone does not provide a prescription for constructing an ensemble. This is in contrast with the classical setting, where a probability distribution (i.e. in the phase space) suffices. However, a density matrix plus a guarantee that the ensemble is a mixture of the pointer states does give such a prescription. Let us consider $\rho_{\mathbf{o}}$ along with the einselected set of states $\{|i\rangle\}$ which emerge as a result of the interaction with the environment. We consider an artificially prepared ensemble $\boldsymbol{a}$ with a density matrix $\rho_{\boldsymbol{a}}$, which we make 'classical by construction'. Ensemble $\boldsymbol{a}$ consists of systems identical to the one described by $\mathbf{o}$. The systems are continuously monitored by an appropriate measuring apparatus which can interact with and keep records of each system in $\boldsymbol{a}$.

Let us first focus on the case of pure decoherence. Then, in the einselected basis,

$$\rho_{\mathbf{o}}(t=0)=\sum_{i,j}\alpha_i^*\alpha_j|i\rangle\langle j| \rightarrow \sum_i |\alpha_i|^2|i\rangle\langle i| = \rho_{\mathbf{o}}(t\gg t_{\mathrm{D}}), \tag{4.1}$$

where $t_{\rm D}$ is the decoherence time-scale. This very same evolution shall occur for both $\rho_{\mathbf{o}}$ and $\rho_{\boldsymbol{a}}$. We can certainly arrange this by adjusting the interactions in the two cases.

In particular, the (pointer) states $\{|i\rangle\}$ shall remain untouched by decoherence.

In the artificial case $\boldsymbol{a}$, the interpretation is inescapable. Each number of $\boldsymbol{a}$ comes with a 'certificate' of its state (which can be found in the memory of the recording device). Following the initial measurement (which establishes the correlation and the first record), the subsequent records will reveal a very boring 'history' (i.e. $|i=17\rangle_{@t_1}, |i=17\rangle_{@t_2}, \ldots, |i=17\rangle_{@t_n}$, etc.). Moreover, the observer, any observer, can remeasure members of $\boldsymbol{a}$ in the basis $\{|i\rangle\}$ and count the number of outcomes corresponding to distinct states of each of the N members. There is no 'collapse' or 'branching' and no need to invoke Born's rule. All of the outcomes are in principle predetermined, as can eventually be verified by comparing the record of the observer with the ongoing record of the monitoring carried out by the measuring devices permanently attached to the system. Individual systems in $\boldsymbol{a}$ have 'certified' states, counting is possible, and, hence, probability can be arrived at through the frequency interpretation for the density matrix $\rho_{\boldsymbol{a}}$. But, at the level of density matrices, $\rho_{\boldsymbol{a}}$ and $\rho_{\mathbf{o}}$ are indistinguishable by construction. Moreover, they have the *same* pointer states, $\{|i\rangle\}_{\mathbf{o}} = \{|i\rangle\}_{\boldsymbol{a}}$. Since all the physically relevant elements are identical for $\mathbf{o}$ and $\boldsymbol{a}$, and since, in $\boldsymbol{a}$, the frequency interpretation leads to the identification of the coefficients of $|i\rangle\langle i|$ with probabilities, the same must be true for the eigenvalues of $\rho_{\mathbf{o}}$.

The 'ignorance' interpretation of the probabilities in $\boldsymbol{a}$ is also obvious. The state of each and every system is registered, but until the 'certificate' for a *specific* system is consulted, its state remains unknown. Similarly, each system in $\mathbf{o}$ can be said to have a state recorded by the environment, waiting to be discovered by consulting the record dispersed between the environmental degrees of freedom. This statement should not be taken too far—the environment is only *entangled* with the system—but it is surprising how little difference there is between the statements one can make about $\mathbf{o}$ and $\boldsymbol{a}$. In fact, there is surprisingly little difference between this situation and the case where the system is completely classical. Consider the familiar Szilard's engine (Szilard 1925), where the observer (Szilard's demon) makes a measurement of a location of a classical particle. The correlation between the particle and the records of the demon can be undone (until or when the demon's record is copied). Thus, 'collapse' may not be as purely quantum as it is usually supposed. And information transfer is at the heart of the issue in both classical and quantum contexts. In any case, our goal here has been a frequentist justification of probabilities. And that goal we have accomplished using a very different approach than those based on the frequency operator attempts to derive Born's formula put forward to date (Graham 1970; Hartle 1968; Fahri *et al*. 1989; Kent 1990).

To apply the strategy of the *standard definition* of probabilities in quantum physics, we must identify circumstances under which possibilities, mutually exclusive 'events', can be permuted without having any noticeable effect on their likelihoods. We shall start with the decoherent density matrix which has all of the diagonal coefficients equal:

$$\rho = N^{-1}\sum_{k=1}^{N}|k\rangle\langle k| = \mathbf{1}. \qquad (4.2)$$

Exchanging any two k obviously has no effect on ρ and, therefore, on the possible measurement outcomes. Thus, when we assume that the total probability is normalized and equal to unity, a probability of any individual outcome $|k\rangle$ must be given by

$$\mathrm{Tr}\,\rho|k\rangle\langle k| = N^{-1}. \tag{4.3}$$

It also follows that a probability of a combination of several (n) such elementary events is

$$\mathrm{Tr}\,\rho(|k_1\rangle\langle k_1| + |k_2\rangle\langle k_2| + \cdots + |k_n\rangle\langle k_n|) = n/N. \tag{4.4}$$

Moreover, when before the onset of decoherence the system was described by the state vector

$$|\psi\rangle = N^{-1/2}\sum_{k=1}^{N} \mathrm{e}^{\mathrm{i}\phi_k}|k\rangle, \tag{4.5}$$

the probabilities of the alternatives after decoherence in the basis $\{|k\rangle\}$ will be

$$p_{|k\rangle} = |\langle k|\psi\rangle|^2 = N^{-1}. \tag{4.6}$$

However, in order to be able to add or to permute different alternatives without any operational implications, we must have assumed decoherence. For, as long as $|\psi\rangle$ is a superposition (equation (4.5)), one can easily invent permutations which will effect measurement outcomes. Consider, for example,

$$|\psi\rangle = (|1\rangle + |2\rangle - |3\rangle)/\sqrt{3}. \tag{4.7}$$

A measurement could involve alternatives $\{|1\rangle, |2\rangle + |3\rangle, |2\rangle - |3\rangle\}$ and would easily distinguish between the $|\psi\rangle$ above and the permuted

$$|\psi'\rangle = (|3\rangle + |2\rangle - |1\rangle)/\sqrt{3}. \tag{4.8}$$

The difference between $|\psi\rangle$ and $|\psi'\rangle$ is the relative phase. Thus, decoherence and a preferred basis with identical coefficients are *both* required to implement the standard definition in the quantum context.

The case of unequal probabilities is dealt with by reducing it to (or at least approximating it by) the case of equal probabilities. Consider a density matrix of the system

$$\rho_{\mathcal{S}} = \sum_{k=1}^{N} p_k|k\rangle\langle k|. \tag{4.9}$$

We note that it can be regarded as an average of an equal probability density matrix of a composite system consisting of $\mathcal{S}$ and $\mathcal{R}$:

$$\rho_{\mathcal{SR}} \cong \sum_{k=1}^{N}\sum_{j=n_k}^{n_{k+1}-1} |k,j\rangle\langle k,j|/M. \tag{4.10}$$

Here M is the total number of states in $\mathcal{H}_{\mathcal{SR}}$, and the degeneracies n_k are selected so that $p_k \simeq n_k/M$, i.e.

$$n_k \cong \sum_{k'=1}^{k} p_{k'}\cdot M, \qquad \sum_{k=1}^{N} n_k = M. \tag{4.11}$$

For sufficiently large M (typically $M \gg N$) 'coarse-grained' $\tilde{\rho}_{\mathcal{S}}$ and $\rho_{\mathcal{S}}$ will become almost identical:

$$\tilde{\rho}_{\mathcal{S}} = \lim_{M/N\to\infty} \sum_{k=1}^{N} \sum_{j=n_k}^{n_{k+1}-1} \langle j|\rho_{\mathcal{SR}}|j\rangle = \lim_{M/N\to\infty} \mathrm{Tr}_{\mathcal{R}}\, \rho_{\mathcal{SR}} = \rho_{\mathcal{S}}. \tag{4.12}$$

One can now use the 'standard' argument to obtain, first, the probability interpretation for $\rho_{\mathcal{SR}}$ (based on the invariance of $\rho_{\mathcal{SR}}$ under the permutations $(kj) \to (k'j')$), and then use it (and equation (4.4)) to deduce the probabilistic interpretation of $\rho_{\mathcal{S}}$. Note that in the above sum (equation (4.12)), we did not have to appeal to the actual numerical values of the eigenvalues of $\rho_{\mathcal{SR}}$, but only to their equivalence under the permutations. Thus, we are not assuming a probabilistic interpretation of $\rho_{\mathcal{SR}}$ to derive it for $\rho_{\mathcal{S}}$. (We also note that the sum over auxiliary states above is, strictly speaking, not a conventional trace since the dimensions of subspaces traced out for distinct k will in general differ. For those concerned with such matters we point out that one can deal with subspaces of equal dimensionality providing that the 'dimension deficit' is made up by auxiliary states which have zero probability.)

This completes the second approach to the quantum probabilities. Again, we have reduced the problem to counting. This time, it was a count of equivalent alternatives (rather than of events). In both of these approaches decoherence played an important role. In the standard definition, decoherence got rid of the distinguishability of the permuted configurations and einselection defined what they were. In the frequency interpretation einselection was essential: it singled out states which were stable enough to be counted and verified.

Our last approach starts from a point of departure which does not rely on counting. Gnedenko was less than sympathetic to the definitions of probability as a measure of a 'degree of certainty', which he regarded as a 'branch of psychology' rather than a foundation of a branch of mathematics. We shall also find our attempt in this direction least concrete of the three, but some of the steps are nevertheless worth sketching.

Gnedenko's discomfort with the 'degree of certainty' might have been alleviated if he had been familiar with the paper of Cox (1946), who, in effect, derived basic formulae of the theory of probability starting from such a seemingly subjective foundation by insisting that the 'measure' should be consistent with the laws of Boolean logic.

Intuitively, this is a very appealing demand. Probability emerges as an extension of the two-valued logic into a continuum of the 'degrees of certainty'. The assumption that one should be able to carry classical reasoning concerning 'events' and get consistent estimates of the conditional degree of certainty leads to algebraic rules which must be followed by the measure of the degree of certainty.

This implies that an information-processing observer who employs classical logic states and classical memory states which do not interfere will be forced to adopt calculus of probabilities essentially identical to what we have grown accustomed to. In particular, the likelihood of c and b (i.e. 'proposition $c{\cdot}b$') will obey a multiplication theorem:

$$\mu(c \cdot b|a) = \mu(c|b \cdot a)\mu(b|a). \tag{4.13}$$

Above, $\mu(b|a)$ designates a conditional likelihood of b given that a is certain. Moreover, μ should be normalized:

$$\mu(a|b) + \mu(\sim a|b) = 1, \tag{4.14}$$

where $\sim a$ is the negation of the proposition a. Finally, the likelihood of c or b ($c \cup b$) is

$$\mu(c \cup b|a) = \mu(c|a) + \mu(b|a) - \mu(c \cdot b|a), \tag{4.15}$$

which is the ordinary rule for the probability that at least one of two events will occur.

In short, if classical Boolean logic is valid, then the ordinary probability theory follows. We are halfway through our argument, as we have not yet established the connection between the μ and the state vectors. But it is important to point out that the assumption of the validity of Boolean logic in the derivation involving quantum theory is non-trivial. As was recognized by Birkhoff & von Neumann (1936), the distributive law $a \cdot (b \cup c) = (a \cdot b) \cup (a \cdot c)$ is *not* valid for quantum systems. Without this law, the rule for the likelihood of the logical sum of alternatives (equations (4.14), (4.15)) would not have held. The physical culprit is quantum interference, which, indeed, invalidates probability sum rules (as is well appreciated in examples such as the double-slit experiment). Decoherence destroys interference between the einselected states. Thus, with decoherence, Boolean logic and, consequently, classical probability calculus with its sum rules are recovered.

Once it is established that 'likelihood' must be a measure (which, in practice, means that μ is non-negative, normalized, satisfies sum rules and that it depends only on the state of the system and on the proposition), Gleason's (1957) theorem implies that

$$\mu(a|b) = \mathrm{Tr}(|a\rangle\langle a|\rho_b), \tag{4.16}$$

where ρ_b is a density matrix of the system, and $|a\rangle\langle a|$ is a projection operator corresponding to the proposition a. Thus, starting from an assumption about the validity of *classical* logic (i.e. absence of interference) we have arrived, first, at the sum rule for probabilities and, subsequently, at Born's formula.

Of the three approaches outlined in this section the two 'traditional' ones are more direct and, at least to this author, more convincing. The last approach is of interest more for its connection between logic and probability than as a physically compelling derivation of probabilities. We have described it in that spirit. These sorts of logical considerations have played an important part in the motivation and the subsequent development of the 'consistent histories' approach (Griffiths 1984, 1996; Gell-Mann & Hartle 1990; Omnès 1992).

5. Relatively objective existence. In what sense is the moon there when nobody looks?

The subjective nature of quantum states is at the heart of the interpretational dilemmas of quantum theory. It seems difficult to comprehend how quantum fuzziness could lead to the hard classical reality of our everyday experience. A state of a classical system can in principle be measured without being perturbed by an observer who knew nothing about it beforehand. Hence, it is said that classical physics allows states to exist objectively. Operationally, when observer A prepares a classical ensemble $\boldsymbol{a}_{\mathrm{c}}$ and hides the list $\mathcal{L}_{\mathrm{A}}^{\mathrm{c}}$ with the records of the state of each system in $\boldsymbol{a}_{\mathrm{c}}$ from the observer B, it will be still possible for B to find out the states of each member of $\boldsymbol{a}_{\mathrm{c}}$ through a measurement, with no *a priori* knowledge. To verify this, B could supply his list $\mathcal{L}_{\mathrm{B}}^{\mathrm{c}}$ for inspection. Classical physics allows $\mathcal{L}_{\mathrm{A}}^{\mathrm{c}}$ and $\mathcal{L}_{\mathrm{B}}^{\mathrm{c}}$ to be always identical. Moreover, both lists will be the same as the new list $\mathcal{L}_{\mathrm{A}}^{\mathrm{c}'}$ with the states of $\boldsymbol{a}_{\mathrm{c}}$ remeasured by A to make sure that $\boldsymbol{a}_{\mathrm{c}}$ was not perturbed by B's measurements. Indeed, it is impossible for A to find out, just by monitoring the systems in the ensemble $\boldsymbol{a}_{\mathrm{c}}$, whether some enterprising and curious B has discovered all that there is to know about $\boldsymbol{a}_{\mathrm{c}}$. Measurements carried out on a classical $\boldsymbol{a}_{\mathrm{c}}$ can be accomplished without leaving an imprint.

This Gedankenexperiment shall be the criterion for the 'objective existence'. When all of the relevant lists match, we shall take it as operational evidence for the 'objective nature of measured states'. In the case of a quantum ensemble $\boldsymbol{a}_{\mathrm{q}}$ this experiment cannot succeed *when it is carried out on a closed system*. Observer A can of course prepare his list $\mathcal{L}_{\mathrm{A}}^{\mathrm{q}}$, a list of Hilbert space states of all the systems in $\boldsymbol{a}_{\mathrm{q}}$. B could attempt to discover what these states are, but in the absence of any prior knowledge about the observables selected by A, one set at a time, in the preparation of each system in $\boldsymbol{a}_{\mathrm{q}}$, he will fail—he will reprepare members of $\boldsymbol{a}_{\mathrm{q}}$ in the eigenstates of the observables he has selected. Hence, unless by sheer luck B elects to measure the same observables as A for each member of $\boldsymbol{a}_{\mathrm{q}}$, $\mathcal{L}_{\mathrm{A}}^{\mathrm{q}}$ and $\mathcal{L}_{\mathrm{B}}^{\mathrm{q}}$ will not match. Moreover, when A remeasures the quantum ensemble using his 'old' observables (in the Heisenberg picture, if necessary) following the measurement carried out by B, he will discover that his new list $\mathcal{L}_{\mathrm{A}}^{\mathrm{q}'}$ and his old list $\mathcal{L}_{\mathrm{A}}^{\mathrm{q}}$ do not match either.

This illustrates the sense in which states of quantum systems are subjective—they are inevitably shaped by measurements. In a closed quantum system it is impossible to just 'find out' what the state is. Asking a question (choosing the to-be-measured observable) guarantees that the answer (its eigenstate) will be consistent with the question posed (Wheeler 1983).

Before proceeding, we note that in the above discussions we have used a 'shorthand' to describe the course of events. What was really taking place should have been properly described in the language of correlations. Thus, especially in the quantum case, the objectivity criterion concerns the correlation between a set of several lists $(\mathcal{L}_{\mathrm{A}}^{\mathrm{q}}, \mathcal{L}_{\mathrm{B}}^{\mathrm{q}}, \mathcal{L}_{\mathrm{A}}^{\mathrm{q}''})$, which were presumably imprinted in effectively classical (i.e. einselected with the help of appropriate environment) sets of record states. The states of the systems in the ensemble $\boldsymbol{a}_{\mathrm{q}}$ played a role similar to the communication channels. The operational definition of objective existence of the state of the system hinges on the ability of the state of the system to serve as a 'template', which can remain unperturbed while it is being imprinted onto the records of the initially ignorant observers. States of isolated quantum systems cannot serve this purpose—they are too malleable! We shall see below that the einselected states of decohering quantum systems are less fragile and can be used as a 'template'. Again, we shall use a shorthand, talking about states, while the real story is played out at the level of multipartite *correlations*. We assume the reader will continue to translate the shorthand into the 'full version'.

Consider $\boldsymbol{a}_{\mathrm{e}}$, an ensemble of quantum systems subject to ideal einselection caused by the interaction with the environment. If the systems are to retain their states in spite of decoherence, the observer A has very little choice in the observables he can use for preparation. The menu of stable correlations between the states of systems in $\boldsymbol{a}_{\mathrm{e}}$ and his records is limited to those involving einselected pointer states. Only such states will be preserved by the environment for periods of time long enough to contemplate the Gedankenexperiment described above.

The task of the observer B (who is trying to become correlated with the stable states of $\boldsymbol{a}_{\mathrm{e}}$ without destroying pre-existing stable correlations established by the observer A) is simplified. As soon as he finds out what the pointer observables are, he can measure at will. He can be sure that, in order to get sets of records with predictive power, A must have selected the same pointer observables to prepare $\boldsymbol{a}_{\mathrm{e}}$. And as soon as the pointer observables are known, their eigenstates can be found without being perturbed. Moreover, B will be measuring observables which are already being monitored by the environment, so his measurements will have no discernible effect on the states of the systems in $\boldsymbol{a}_{\mathrm{e}}$.

Hence, either A was smart enough to choose pointer states (in which case his lists $\mathcal{L}_{\mathrm{A}}^{\mathrm{e}}, \mathcal{L}_{\mathrm{A}}^{\mathrm{e}'}, \ldots$ will all be identical) and B's spying will not be detected, *or* A chooses to measure and prepare arbitrary states in the Hilbert space, guaranteeing their deterioration into mixtures of pointer states at a rapid decoherence rate. In this second case, A's lists will reflect a steady increase in entropy caused by the mismatch between the observable he has elected to measure and the pointer observables he should have measured, and B's spying will most likely still be undetected (especially if he is smart enough to focus on the pointer observables).

Let us now compare the three variants of the Gedankenexperiment above. In the classical case, anyone can find out states of the systems in $\boldsymbol{a}_{\mathrm{c}}$ without perturbing them. Prior information is unnecessary, but only classical (i.e. localized, etc.) states can be used. In the case of a quantum isolated system an enormous variety of quantum states, including all conceivable superpositions of the classical states, can be prepared by A in $\boldsymbol{a}_{\mathrm{q}}$. Now B's measurement will almost inevitably reprepare these states, unless somehow B knows beforehand what to measure (i.e. what observables A has measured). In the third case—quantum, but with decoherence and einselection—the choices of A are limited to the preferred pointer states. Only a very small subset of all the superpositions in the Hilbert space $\mathcal{H}$ is available. Moreover, the environment is already carrying out continuous monitoring of the observables it has elected to 'measure'. B can use the correlations established between the system and the state of the environment to find out what the preferred observables are and to measure them. He will of course discover that his list matches A's lists and that A could not have detected B's 'spying'.

When the states can be 'revealed' without being reprepared in the process, they can be thought to exist objectively. Both the classical case and the quantum plus einselection case share this feature. The environment-induced superselection simultaneously decreases the number of states in $\mathcal{H}$ while allowing the einselected states to 'exist objectively'—to be found out without the necessity of repreparing them in the process.

In fact, the measurements we carry out as observers are taking an even more immediate advantage of the monitoring carried out by the environment. Our measurements are almost never direct—nearly without exception they rely on interception of the information already present in the environment. For instance, all of the visual information we acquire comes from a tiny fraction of the photon environment intercepted by the rod and cone cells in our eyes. Indeed, this is perhaps the best strategy

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observer B could have used in the third version of the Gedankenexperiment above. Rather than directly interacting with the system in $\boldsymbol{a}_{\mathrm{e}}$, he could have monitored the environment. An imprint left in a small fraction of its state is usually enough to determine the state of the system; the environment contains a vastly redundant record of the pointer observables (Zurek 1983). Thus, perception of classical reality seems inevitable for the observers who, like us, rely on the second-hand information, on the correlations acquired indirectly from the environment.

In a sense the environment plays the role of a commonly accessible 'Internet-like' database which allows one to make copies of the records concerning the state of the system. There is no need to measure the system directly; it suffices to consult the relevant 'Web page', and there is no danger of altering the state of the system: non-separability and other such manifestations of quantum theory could reappear only if, somehow, all of the widely disseminated copies of the information were captured and processed in the appropriate (quantum) manner. The difficulty of such an enterprise in the macroscopic domain (which we have quantified before by the redundancy distance (equations (3.6)–(3.12)) is a measure of the irreversibility of the decoherence-induced 'reduction of the wavepacket'.

We have just established that states of quantum systems interacting with their environments exist much like the classical states were presumed to exist. They can be *revealed* by measurements of the pointer observables which can be ascertained without prior knowledge. In particular, indirect measurements—observations monitoring the environment in search of the imprints of the state of the system—seem to be the safest and at the same time the most realistic way to reveal that state. Moreover, there are many fewer possible einselected states than there are states in the Hilbert space. Thus, the relative objectivity based on the system–environment correlations and, hence, on decoherence and einselection, comes at a price: the choice is severely limited†.

In the above operational approach to the definition of existence, we have made several simplifying assumptions. We have (i) neglected the evolution; (ii) assumed perfect decoherence; and (iii) focused on observers with 'perfect knowledge', i.e. used pure states rather than mixtures as initial conditions. All of these assumptions can be relaxed with relatively little pain. Hamiltonian evolution *alone* would not be a problem—the system could be described in Heisenberg's picture. But the combination of evolution and decoherence will lead to complications, resulting in a preferred basis which is imperfect (Zurek *et al*. 1993*a*; Tegmark & Shapiro 1994; Gallis 1996; Anglin & Zurek 1996; Anglin *et al*. 1997)—even the optimal pointer states would

† How limited? There are of course infinitely many superpositions in $\mathcal{H}$ of finite dimensionality, but that is already true of a spin-$\frac{1}{2}$ Hilbert space, and it does not capture the reason for the incredible proliferation of superpositions. In the Hilbert space of a decohering system, there will be $N \sim \dim(\mathcal{H})$ pointer states $\{|k\rangle\}$. For a typical superposition state $|\psi\rangle$ composed of all N states, with the possibilities 'coarse grained' by assuming that all $|\psi\rangle$ have a form

$$|\psi\rangle = \frac{1}{\sqrt{N}} \sum_k (-)^{i_k} |k\rangle,$$

where i_k is 0 or 1, there will be

$$\mathcal{W} \sim 2^N$$

such superpositions. That is, even when we set all the coefficients to have the same absolute value, and coarse-grain phases to the least significant (binary) digit, we will have exponentially many possibilities.

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eventually deteriorate into mixtures, albeit on a time-scale that is long compared to the decoherence time-scale for random superposition in $\mathcal{H}$.

This difference between the einselected states and arbitrary superpositions could be quantified by defining the predictability horizon

$$t_{\mathrm{p}} = \int_0^\infty \frac{(H_{\mathrm{eq}} - H(t))}{(H_{\mathrm{eq}} - H(0))}\,\mathrm{d}t. \tag{5.1}$$

This characterizes the time-scale over which the initial information $H_{\mathrm{eq}} - H(0)$ is lost as the von Neumann entropy, $H(\rho) = -\operatorname{Tr}\rho\log\rho$, approaches the long-term (equilibrium) value H_{eq}. Easier to compute (and similarly motivated),

$$t'_{\mathrm{p}} = \int_0^\infty \operatorname{Tr}(\rho_{\mathrm{t}}^2 - \rho^2(\infty))\,\mathrm{d}t \tag{5.2}$$

should supply equivalent information. Thus t'_{p} would be short (and of the order of the decoherence time) for a typical initial state in the Hilbert space. By contrast, the predictability horizon may be long (and, perhaps, even infinite) for pointer states of integrable systems, while for chaotic systems one would anticipate predictability time-scales determined by the Lyapunov exponents when decoherence dominates (Zurek & Paz 1994, 1995).

The Gedankenexperiment at the foundation of our 'operational definition of existence' is based on the comparison of records of two observers A and B. It could now be repeated, provided that the duration of the experiment is brief compared to the predictability time-scale, or that the natural rate of information loss is accounted for when evaluating the final results.

In fact, the predictability sieve could be implemented by using this strategy. Einselected pure states will maximize t_{p}. Moreover, such a procedure based on the predictability time-scale can be easily applied to compare pure and mixed states. That is, one can find out how much more durable various correlations between the observer's records of the coarse-grained measurements are. The key difference from the original predictability sieve (Zurek 1993*a*) which has been successfully used to demonstrate the special role of Gaussians (Zurek 1993*a*; Zurek *et al*. 1993; Tegmark & Shapiro 1994; Gallis 1996) is a somewhat different sieve criterion, which may even have certain advantages.

All these caveats and technicalities should not obscure the central point of our discussion. Environment-induced superselection allows observers to anticipate what states in the Hilbert space have a 'relatively objective existence' and can be revealed by measurements without being simultaneously reprepared. *Relatively objective existence* is a deliberate double entendre, trying to point out both the relative manner in which existence is defined (i.e. through correlations, similar in spirit to the relative states of Everett (1957)) and a reminder that the existence is not absolutely stable but, rather, that it is purchased at the price of decoherence and based on the monitoring by the environment.

Concerns about the predictability time-scale do not imply that, on time-scales that are long compared to t_{p}, the states of the systems in question do not 'exist'. Rather, t_{p} indicates the predictability horizon on which evolution and decoherence destroy the relevance of the 'old' data (the record-state correlation). But even then the essence of our definition of reality—the ability of the observer to 'reveal' the state—captures the essence of 'existence'.

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6. The existential interpretation

The interpretation based on the ideas of decoherence and einselection has not really been spelled out to date in any detail. I have made a few half-hearted attempts in this direction (Zurek 1993*a*, *b*), but, frankly, I was hoping to postpone this task, since the ultimate questions tend to involve such 'anthropic' attributes of the 'observership' as 'perception', 'awareness' or 'consciousness', which, at present, cannot be modelled with a desirable degree of rigour.

It was my hope that one would be able to point to the fact that decoherence and einselection allow for existence (as defined operationally through relative states and correlations in the preceding section) and let those with more courage than I worry about more esoteric matters. I have been gradually changing my mind as a result of penetrating questions from my colleagues and the extrapolations (attempted by the others) of the consequences of decoherence and einselection which veered in directions quite different from the ones which I had anticipated (see Elby (1993), Healey (1998), Bub (1997) and d'Espagnat (1995) for a sample of questions, criticisms, and attempts at an interpretation). Moreover, while there are 'deep' questions involving consciousness which may be too ambiguous to attack with the tools used by physicists, there are aspects of information processing which bear direct relevance for these deeper issues, and which can be analysed profitably in a reasonably concrete setting. Here I intend to amplify some of the points I have made before and to provide the 'next iteration' by investigating the consequences of environmental monitoring a step or two beyond the operational definition of 'existence'. I shall proceed in the general direction indicated earlier (Zurek 1993*a*, *b*), and focus on the stability of the einselected correlations.

We start by noting that the relatively objective existence of certain states 'negotiated' with the environment has significant consequences for the observers and their information-processing abilities. In the Gedankenexperiments involving observers A and B in the preceding section we could have equally well argued for the objective existence of the states of their memory cells. Again, superpositions of all the possibilities are ruled out by einselection, and the brain of an observer can preserve, for long periods of time, only the pointer states of its neurons. These states exist in the same relatively objective sense we have defined before—they can reveal (correlate with) the states of other neurons without having to be simultaneously reprepared. Indeed, real neurons are coupled very strongly to their environments and certainly cannot exist in superpositions. Their two stable states are characterized by different rates of firing, each a consequence of a non-equilibrium dissipation-dominated phenomenon, which are bound to leave a very strong imprint on the environmental degrees of freedom not directly involved in the information processing. In an obviously overdamped system operating at a relatively high temperature, the inability to maintain superpositions is not a surprise†.

When we assume, as seems reasonable, that the states of neurons are the 'seat' of memory and that their interactions lead to information processing (which eventually

† Neurons work more like a diode or a transistor, relying on two stable steady states (characterized by different firing rates) for stability of the logical '0' and '1', rather than the two-state spin-$\frac{1}{2}$-like systems, which often end up playing the neuron's roles in models of neuron networks invented by theoretical physicists. I believe, however, that for the purpose of the ensuing discussion this distinction is not essential and I will continue to invoke *licentia physica theoretica* and consider spin-$\frac{1}{2}$-like neurons for the remainder of this paper.

results in 'awareness', and other such 'higher functions'), we have tangible hardware issues to analyse. Indeed, at this level of discussion there is little fundamental difference between a brain and a massively parallel neural network-like *effectively classical* computer.

The ability to process information concerning states of objects external to memory (for, say, the purpose of prediction) is then based on the stable existence of correlations between the record bits of memory and the state of the object. It is fairly easy to imagine how such a correlation can initially be established either by a direct measurement or, as we have noted previously, through an indirect process involving the environment. For the reliability of memories, it is absolutely crucial that this correlation be immune to further correlations with the environment, i.e. to decoherence. Following a measurement (and the initial bout of decoherence), the reduced joint density matrix of the system and the relevant part of memory and the environment will have the form

$$\rho_{\mathcal{SM}} = \sum_i p_i |s_i\rangle\langle s_i||\mu_i\rangle\langle\mu_i|. \tag{6.1}$$

The predictability horizon can be defined as before through

$$t_{\mathrm{p}}^{(i)} = \frac{\int_{\mathrm{p}}^{\infty}(H(s_i,\mu_i;t) - H(s_i,\mu_i;\infty))\,\mathrm{d}t}{H(s_i,\mu_i;0) - H(s_i,\mu_i;\infty)} \tag{6.2}$$

for individual outcomes. Here H can stand for either Shannon–von Neumann, or linear (or still other) measures of information content of the conditional (re)normalized $\langle\mu_i|\rho_{\mathcal{SM}}|\mu_i\rangle$. After a perfect measurement there is a one-to-one correlation between the outcome $\mathcal{S}_i$ and the record μ_i (equation (6.1)). It will, however, deteriorate with time as a result of the dynamics and the openness of the system, even if the record-keeping memory states are perfectly reliable (Zurek 1998). The predictability time-scale for memory-system joint-density matrices has a more specific interpretation than the one defined by equations (5.1) and (5.2). It is also safe to assume that memories use stable states to preserve records. In this 'no amnesia' case, $t_{\mathrm{p}}^{(i)}$ will measure the time-scale on which the acquired information is becoming useless 'old news' because of the unpredictable evolution of the open system. The predictability horizon can (and typically will) depend on the outcome.

We note that more often than not, both the states of memory and the states of the measured systems will be mixed coarse-grained states inhabiting parts of large Hilbert spaces, rather than pure states. Thus the record μ_i will correspond to ρ_{μ_i} and $\mathrm{Tr}(\rho_{\mu_i}\rho_{\mu_j}) \cong \delta_{ij}$. It is straightforward to generalize equations (6.1) and (6.2) to cover this more realistic case. Indeed, it is likely that the memory states will be redundant, so that the likely perturbations to the 'legal' memory states will retain orthogonality. This would allow for classical error correction, such as is known to be implemented, for example, in neural circuits responsible for photodetection in mammalian eyes, where several (approximately seven) rods must fire more or less simultaneously to result in a record of detection of a light source.

Observers will be able to make accurate predictions for as long as a probabilistic equivalent of logical implication is valid, that is, as long as the conditional probability $g(t)$ defined by

$$p(\sigma_i(t)|\mu_i) = p(\sigma_i(t),\mu_i)/p(\mu_i) = g(t) \tag{6.3}$$

is close to unity. Here $\sigma_i(t)$ is a 'proposition' about the state of the system at a time t. One example of such a formal equivalent of a proposition would be a projection operator onto a subspace of a Hilbert space. When $\sigma_i(t)$ is taken to be a pure state $|s_i(t)\rangle$, and $t \ll t_{\mathrm{p}}^{(i)}$ so that $g(t) \cong 1$, equation (6.3) becomes in effect a formal statement of the 'collapse' axiom of quantum measurement theory. For memory will continue to rediscover the system in the same state upon repeated remeasurements.

Again, as in the preceding section, relatively objective existence of *correlations* (established in contact with the environment, with their stability purchased at the expense of the limitation of the possible states of memory and the measured system) is decisive for the predictive utility of the records. These records must be repeatedly and reliably accessible for the other parts of memory to allow for information processing. This is why the record states which *exist* (at least in the relatively objective operational sense employed before) are essential for the reliability of memories inscribed in open systems. They can be remeasured by the other memory cells, spreading the useful correlation throughout the information-processing network of logical gates but suffer no ill effects in the process.

The record state $|\mu_i\rangle$ must then obviously be decoherence resistant, but the same should be true for the measured states $|s_i(0)\rangle$ and (hopefully) for their near-future descendants $|s_i(t)\rangle$. Only then will the correlation between memory and the state of the system be useful for the purpose of predictions. One can analyse this persistence of quasi-classical correlations from various points of view, including the algorithmic information content one. We shall mention this idea here only briefly as a more complete account is already available (Zurek 1998). In essence, when $|s_i(t)\rangle$ evolves predictably, a sequence of repeated measurements of the appropriate observables yields a composite record $R = \{\mu^{(1)}_{@t_1}, \mu^{(2)}_{@t_2}, \dots, \mu^{(n)}_{@t_n}\}$, which will all be derivable from the initial $\mu^{(0)}_{@t_0}$ and from the algorithm for the evolution of the monitored system. This predictability could be expressed from the viewpoint of the observer by comparing the algorithmic information content of R with its size in bits. When the whole R can be computed from the initial condition, the algorithmic information content $K(R)$ (defined as the size of the shortest program for a universal computer with the output R (Li & Vitányi 1993)) is much less than the size of the 'uncompressed' R in bits. An illustrative (if boring) example of this would be a sequence of records of a fixed state of an object such as a stone. Then R would simply be the same record, repeated over and over. This is of course trivially algorithmically compressible.

This immobility of objects such as stones is the basic property which, when reflected in the perfectly predictable sequence of records, provides a defining example of (the most basic type of) perception of existence, of permanence which defines 'classical reality'. In this case, the same set of observables 'watched' by the environment is also being watched by the observer.

In general, the state of a system evolving in contact with the environment will become less and less closely correlated with its initial state. Therefore, entropy will increase, and the reliability of the implication measured by the conditional probability $p(\sigma_i(t)|\mu_i)$ will decrease. Alternatively, one may want to retain the safety of predictions (i.e. have $p(\sigma_i(t)|\mu_i)$ close to unity at the price of decreased accuracy). This could be accomplished by choosing a safer (but less precise) prediction $\tilde{\sigma}_i(t)$ which includes $\sigma_i(t)$ with some 'error margin' and thus supplies the requisite redundancy.

For a judiciously selected initial measurement, the conditional probability $p(\sigma_i(t)|\mu_i)$ will decrease relatively slowly (for example, on a dynamically determined Lyapunov time-scale in the case of chaotic systems (Zurek 1998)), while for measurements which result in the preparation of a 'wrong' initial condition—a state at odds with einselection—the conditional probability would diminish suddenly, on a near-instantaneous decoherence time-scale.

Typically, the prediction σ_i will not be a pure state but a suitably macroscopic patch in the phase space (and a corresponding 'chunk' of the Hilbert space). Increase in the size of the patch will help extend the relative longevity of the predictive power of the records at the expense of accuracy. Nevertheless, even in this case predictive power of the old records shall eventually be lost with time.

The memory must be stored in robust record states, which will persist (ideally forever, or at least until they are deliberately erased). Deliberate erasure is an obvious strategy when the records outlive their usefulness.

In this picture of a network of effectively classical memory elements interconnected with logical gates, stability of the records is essential. It is purchased at the price of 'censorship" of the vast majority of all of the superpositions which are in principle available in the Hilbert space. It is in an obvious contrast with quantum computers (Feynman 1986; Deutsch 1985; Lloyd 1993; DiVincenzo 1995; Bennett 1995; Williams & Clearwater 1997), where all of the superpositions are available for information processing, and where the states of memory are unstable and prone to the environment-induced decoherence and other errors.

Let us now consider how such a quasi-classical environmentally stable memory 'perceives' the universe. To avoid generalities, we consider, from the point of view of this memory, the task of determining what the classical branches of the universal state vector are. That is, we shall ask the observer to find out what the pointer states in his branch of the universe are.

In effect, we have already presented all of the elements necessary for the definition of the branches in the course of the discussion of existence in the preceding section. A branch is defined by its predictability, by the fact that the correlations between its state and the records of the observer are stable. In other words, a branch is defined by the fact that it does not split into branches as rapidly as a randomly selected state.

The observer is 'attached' to the branch by the correlations between its state and the einselected states which characterize the branch. Indeed, the observer is a part of his branch. In the case of perfect predictability (no entropy production—initial records of the observer completely determine the future development of the branch), there would be no further branching. An observer will then be able, on the basis of his perfect predictability, to develop a view that the evolution of the universe is completely deterministic and that his measurements either confirm perfect predictability (when carried out on the system he has already measured in the past) or help reveal the pre-existing state of affairs within 'his' branch.

This classically motivated, and based on Newtonian intuitions, 'single-branch' (single-trajectory) limit of quantum physics is responsible for the illusion that we live in a completely classical universe. It is an excellent approximation in the realm of the macroscopic, but it begins to fail as the resolution of the measurements increases. One example of failure is supplied by quantum measurements and, more generally, by the situations where the state of the macroscopic object is influenced by the state of a quantum object. Then the past initial condition is exhaustive—the observer knows a pure state, i.e. all that is possible to know—and yet this is demonstrably insufficient to let him determine the future development of his branch. For, when the past correlation is established in a basis incompatible either with the monitoring carried out by the environment, or when it prepares a state which is not an eigenstate of the new measurement, the new outcome cannot be inferred from the old initial condition.

Relatively objective existence is at the core of the definition of branches. Stability of the correlations between the state of the observer and the branch is responsible for the perception of classicality. Stability of the record states of the observer is an obvious precondition. The observer with a given set of records is firmly attached to the branch which has induced these records—he 'hangs on' to the branch by the correlations. He may even be regarded as a part of that branch, since mutual correlations between systems define branches.

Observers described here are quite different from the aloof observers of classical physics, which simply 'note' outcomes of their measurements by adding to their abstract and immaterial repository of information. In a quantum universe *information is physical* (Landauer 1991)—there is simply *no information without representation* (Zurek 1993*b*). In our context this implies that an observer who has recorded one of the potential outcomes is *physically distinct* from an observer who has detected an alternative outcome. Moreover, these two states of the observer are objectively different—they can be 'revealed' from the outside (i.e. by monitoring the environment in which record states are immersed) without disturbing the observer's records.

The question 'why don't we perceive superpositions?' (which has also been repeated by some of those who investigate and support ideas of decoherence (Omnès 1994)) has a straightforward answer. The very physical state of the observer and, thus, his *identity* is a reflection of the information he has acquired. Hence, the acquisition of information is not some abstract physically insignificant act, but a cause of reshaping of the state of the observer. An exaggerated example is afforded by the famous case of Schrödinger's cat (Schrödinger 1935). A cat that dies as the result of an amplified quantum event will certainly be easily distinguishable from the cat that lives on (and can continue observations). Similarly, an effectively classical computer playing the role of an observer will be measurably distinct depending on what outcome was recorded in its data bank.

Coherent superpositions of two memory states will disappear on the decoherence time-scale in the presence of the environment. Hence, a coherent superposition of two distinct identities of an observer does not exist in the relatively objective operational sense introduced previously. Even in the rare cases when a memory bit of an observer enters into a bona fide entanglement with an isolated quantum system, decoherence will intervene and turn that entanglement into an ordinary classical correlation in the basis defined by the einselected record states.

The interpretation which recognizes that decoherence and environment-induced superselection allow for the *existence* of states at the expense of the superposition principle is known as the *existential interpretation* (Zurek 1993*a*, *b*). It accounts for the inability of the observers to 'perceive' arbitrary superpositions. The (relatively) objective existence of the records is a precondition for their classical processing and, therefore, for perception†.

† It is amusing to speculate that a truly quantum observer (i.e. an observer processing quantum

It is easy to design logical states which would distinguish between objectively existing records and accidental states. Redundancy is the key. Thus, when an entanglement between a (two-state) system and a memory cell develops,

$$|\phi_{\mathcal{S}\mu}\rangle = \alpha|\uparrow\rangle|1\rangle + \beta|\downarrow\rangle|0\rangle, \tag{6.4}$$

and, under the influence of the environmental decoherence, rapidly deteriorates to a classical correlation

$$\rho_{\mathcal{S}\mu} = |\alpha|^2|\uparrow\rangle\langle\uparrow||1\rangle\langle 1| + |\beta|^2|\downarrow\rangle\langle\downarrow||0\rangle\langle 0|, \tag{6.5}$$

the reliability of the record state in an arbitrary basis can in principle be tested by the other parts of the memory. Repeated measurements of the same memory cell in different bases and comparing longevity of the state in the $\{|0\rangle, |1\rangle\}$ basis with the (lack of) longevity of the state in the $\{|+\rangle, |-\rangle\}$ basis would do the trick. Thus, as a consequence of decoherence and einselection,

$$\rho_{\mathcal{S}\mathcal{M}} = |\alpha|^2|\uparrow\rangle\langle\uparrow||1\rangle\langle 1||1'\rangle\langle 1'||1''\rangle\langle 1''|\cdots + |\beta|^2|\downarrow\rangle\langle\downarrow||0\rangle\langle 0||0'\rangle\langle 0'||0''\rangle\langle 0''|\ldots \tag{6.6}$$

in the transparent notation, while in the case of measurements of $\{|+\rangle, |-\rangle\}$ record states, there would be no correlation between the consecutive measurements carried out at intervals exceeding the decoherence time-scale. Instead of the two branches of equation (6.6), there would be 2^N branches, where N is the number of two-state memory cells implicated, and a typical branch would be algorithmically random, easily betraying unreliability of the $\{|+\rangle, |-\rangle\}$ record states.

This simplistic model has no pretense to realism. Rather, its aim is to demonstrate a strategy for testing what is reliably known by the observer. A slightly more realistic model would entail redundant records we have mentioned already (equations (3.6)–(3.12)). Thus, the initial correlations would involve several (n) memory cells:

$$\rho_{\mathcal{S}\mu^n} = |\alpha|^2|\uparrow\rangle\langle\uparrow||1\rangle\langle 1|_1|1\rangle\langle 1|_2\ldots|1\rangle\langle 1|_n + |\beta|^2|\downarrow\rangle\langle\downarrow||0\rangle\langle 0|_1|0\rangle\langle 0|_2\ldots|0\rangle\langle 0|_n. \tag{6.7}$$

Then the reliability of the records can be tested by looking for the basis in which all of the records are in accord. This can be accomplished without destruction of all of the original redundant correlation between some of the records and the system.

These toy models establish that, in the presence of decoherence, it is possible to record and that it is possible to find out what information is reliable (which correlations are immune to decoherence). Again, we emphasize that the above toy models have no pretense to a close kinship with real-world observers†.

7. Conclusion

What we have described above is a fairly complete sketch of the physics involved in the transition from quantum to classical. Whether one would now claim that the emerging picture fits better Bohr's 'Copenhagen' framework or Everett's 'many worlds' interpretation seems to be a semantic rather than a substantial issue. To

information in a quantum computer-like fashion) might be able to perceive superpositions of branches which are inaccessible to us, being limited in our information processing strategies to the record states 'censored' by einselection.

† Indeed, one could argue that if some unhappy evolutionary mutation resulted in creatures which were bred to waste their time constantly questioning the reliability of their records, they would have become nourishment for other more self-assured creatures which did not need to pose and settle such philosophical questions before making a useful prediction.

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begin with, decoherence was *not* a part of either of these interpretations. Thus, what we have presented here is clearly beyond either CI or MWI.

The existential interpretation owes Bohr the central question which was always implicit in the early discussions. This question, about the location of the quantum–classical border, is really very similar to questions about 'existence'. We have posed and settled these questions operationally and, thus, provided a quantum justification for some of the original CI program.

On the other hand, we owe Everett the observation that quantum theory should be the key tool in the search for its interpretation. The question and concern may be traced to Bohr, but the language of branches and the absence of explicit collapses and *ab initio* classicality are very much in tune with MWI.

We believe that the point of view based on decoherence settles many of the questions which were left open by MWI and CI. This includes the origin of probabilities as well as the emergence of 'objective existence', although more needs to be done.

In particular, one issue which has often been taken for granted is looming large, as a foundation of the whole decoherence programme. It is the question of what the 'systems' which play such a crucial role in all the discussions of the emergent classicality are. This issue was raised earlier (Zurek 1982, 1983), but the progress to date has been slow at best. Moreover, replacing 'systems' with, say, 'coarse grainings' does not seem to help at all—we have at least tangible evidence of the objectivity of the existence of systems, while coarse grainings are completely 'in the eye of the observer'.

It should be emphasized that reliance on systems does not undermine the progress achieved to date in the study of the role of decoherence and einselection. As noted before (Zurek 1993*a*), the problem of measurement cannot even be stated without a recognition of the existence of systems. Therefore, our appeal to the same assumption for its resolution is no sin. However, a compelling explanation of what the systems are—how to define them given, say, the overall Hamiltonian in some suitably large Hilbert space—would undoubtedly be most useful.

I thank Chris Jarzynski, Michael Nielsen and Chris Zalka for comments on the manuscript.

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